GOMPERTZ CURVE IN PHYSIOLOGY : AN APPLICATION

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Summary : Gompertz curve has been successfully used to estimate mortality intensities, recovery and relapse rates for human beings suffering from specific diseases. Perhaps, no attempt has yet been made to describe statistically the phenomenon of thermoregulation efficiency of man. This paper deals with the statistical approach for describing the above mentioned phenomenon. For this purpose, Gompertz curve has been fitted to the data of recovery palm skin temperature of human subjects, which was collected after removing the hand of a subject dipped for two minutes in cold water maintained at 10±1°C on seven environmental situationsi ncluding altitude. The coefficients of correlation between observed and estimated palm skin temperature expressed logarithmically were close to 0·99 for each situation indicating the high precision of the fitted curve. The average rates of recovery of palm skin temperature have also been compared between these seven environmental situations using analysis of covariance technique. It is concluded that recovery of palm skin temperature of sea level residents during stay at plains is much faster than their recovery at high altitude situations.

Key words : gompertz curve thermoregulation efficiency palm skin temperature

INTRODUCTION

The application of statistical methods to physiological problems is on the increase for a better comprehension and general application of the result. Gompertz curve has been frequently applied in biology. Winsor (22) has suggested its application to the relative growth rate of an organism decreasing exponentially with time. Shock (18) made use of this curve to establish the relationship between physical activity and the rate of ageing. Storer (19) used it to correlate the radiation resistance with age in normal and irradiated populations of mice. Kimball (10) applied this curve for the estimation of mortality intensities in animal experiments and Deming (6) for describing human growth during the adolescent cycle. The applications of Gompertz curve have also been made by several other workers (4,5,7,11,13,17) to solve the practical problems arising in applied biology. Perhaps the application of this curve has not been made to the studies pertaining to thermoregulation efficiency of man. In the present paper an attempt has been made to explain the recovery pattern of palm skin temperature in man (Criteria of thermoregulation efficiency) after removing the hand dipped for 2 min in cold water maintained at 10±1°C, by Gompertz curve The curve has been fitted to data collected at different environmental situations including altitude. The average rates of recovery are compared and discussed for different situations. The superiority of the present fitted curve over the logistic curve has also been proved.

Physiological data and statistical analysis :

The physiological data from the study of Rai *et al.* (15), who observed the recovery pattern of palm skin temperature for 20 min after removal of the hand dipped for 2 min in cold water maintained at 10±1°C in different environmental situations, have been utilized. In their studies only in six subjects palm skin temperature recovered to the original value in 20 min out of 20 subjects studied at sea level at ambient temperatures of 28 and 21°C, 10 subjects at high altitude during their stay at 1st, 2nd and 3rd week at 21°C, again return to sea level at 21°C and in high altitude natives at altitude at 21°C

On plotting the average palm skin temperature against recovery time, a curve somewhat of type S' letter was observed. Thus, logistic as well as Gompertz curves were fitted to explain this phenomenon. Due to better efficiency of the Gompertz curve based on the criteria of residual sum of squares, absolute percentage variation (21) and coefficient of determination (R^2%) the recovery pattern of palm skin temperature for each environmental condition was explained more precisely by Gompertz curve. The equation for the Gompertz curve is $Y = kA^{Bx}$ which may be put in logarithmic. form

$$\log Y = \log k + (\log A)\ B^x \text{ or}$$

or

$$y = a + b\ r^x \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

where $y = \log Y$, $a = \log k$, $b = \log A$, $r = B$.

The response y is linearly related to the independent variate transformed to r^x. If the metameter y is equally variable at each level of x, both approximate and maximum likelihood (ML) estimates of the parameters of the curve may be obtained. The parameters were estimated using standard statistical procedures (2). The method suggested by Rao (16) has been used to calculate the average rates of recovery in different environmental situations and analysis of variance technique has been used to test whether the average rates of recovery are similar in all environmental conditions eliminating the effect of initial value of palm skin temperature (measured after cessation of cold stress) by the procedure of analysis of covariance (3).

RESULTS

The Gompertz curve was fitted to the data of palm skin temperature of those subjects in whom the recovery was completed within twenty minutes. The Gompertz equations were fitted to each situation and the coefficients of correlation between observed and estimated palm skin temperature expressed logarithmically were calculated for these environmental situations (Table I). It is evident that these coefficients of correlation for all environmental conditions are of the order of 0.99 indicating the high precision of Gompertz curve to explain the recovery pattern of palm skin temperature. The graphical representation of recovery pattern of palm skin temperature after removal of hand from cold water is depicted in Fig. 1, where the fitted curves are shown by solid lines and the observed points are indicated by different symbols to represent different situations. Obviously the observed points are lying on the fitted curves indicating again the high precision of the chosen curves. It has been found that average rates of recovery of sea level residents at 28°C during stay at plains differs significantly from other situations at altitude and 21°C at plains except from return to sea level conditions. The adjusted average rates of recovery of sea level residents during stay at plains in

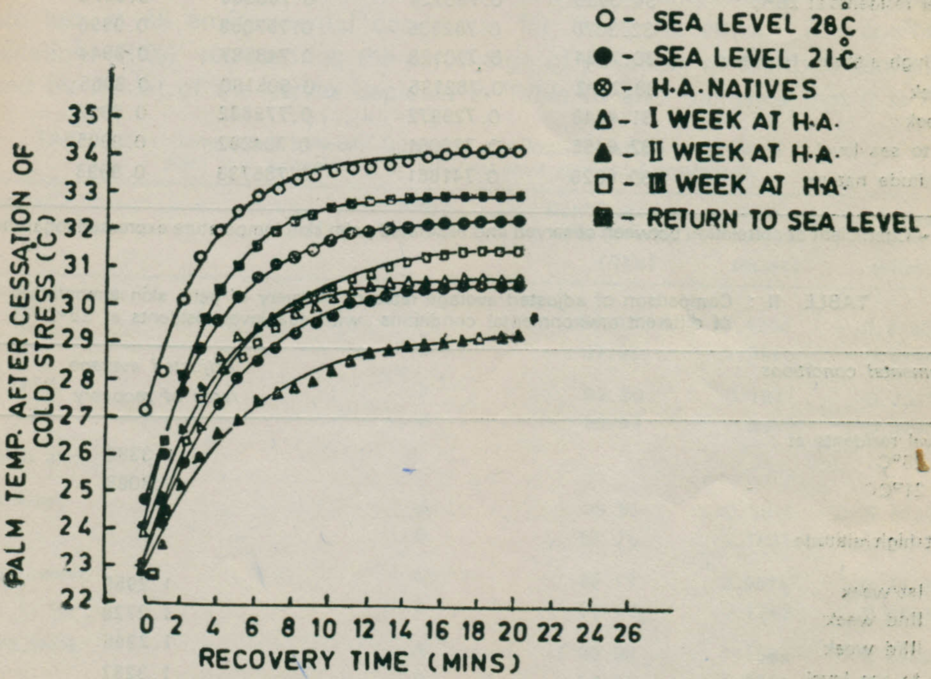

Fig. 1 : Relationship between palm temperature after cessation of cold stress and recovery time.

the ambient temperatures at 28°C and 21°C and thereafter during 1st, 2nd and 3rd week of acclimatization at high altitude and on return to sea level as well as of high altitude natives are shown in Table II. Obviously the adjusted average rates of recovery of sea level residents during their stay at plains in the ambient temperature at 28°C is much faster than the average rates of recovery at high altitude situations as well as to high altitude natives indicating the better thermoregulation efficiency of human subjects at sea level. This finding is in agreement with the results conjectured by several well established workers in applied physiology.

TABLE I : Gompertz equations for different environmentalc onditions to explain recovery pattern of palm skin temperature (Y) with time (X) after cessation of cold stress.

Environmental conditions	*Gompertz Equation* $Y = kA^{BX}$ K	A	B	R	*ML correction*
Sea level residents at 28°C	34.0836	0.790724	0.733839	0.9972	0.00000
21° C	32.3076	0.762936	0.757069	0.9990	0.00001
Stay at high altitude, Ist week	30.7234	0.770128	0.743587	0.9944	0.00001
IInd week	29.2772	0.782195	0.805180	0.9965	0.00000
IIIrd week	31.4648	0.729972	0.779632	0.9952	0.00003
Return to sea level	32.9455	0.726061	0.704092	0.9995	0.00000
High altitude natives	30.6829	0.741681	0.785733	0.9993	0.00001

R – Coefficient of correlation between observed and estimated palm skin temperature expressed logarithmically.

TABLE II : Comparison of adjusted average rates of recovery of palm skin temperature of different environmental conditions with sea level residents at 28°C.

Environmental conditions	*Adjusted average rates of recovery*	*Level of significance*
Sea level residents at		
28°C	1.3380	
21°C	1.3098	*
Stay at high altitude		
Ist week	1.2959	***
IInd week	1.2729	***
IIIrd week	1.2965	***
Return to sea level	1.3237	NS
High altitude natives	1.2831	***

NS – Not significant; * — $P<0.05$; *** — $P<0.001$

A different and delayed recovery rate in case of sea level residents at altitude, indicates that there is reduction in the thermoregulatory efficiency of man, atleast during the initial phases of induction to high altitude This possibly could be due to the direct effect of hypoxia on the neurons of the centre in the hypothalamus responsible for it and the physiochemical effects of the respiratory centre in the brain on the composition of blood passing through hypothalamic area (20). As changes in autonomic nervous system also play an important role in the mammalian thermoregulation (1,14), the relative hyperactivity of sympathetics (12) would also contribute to these thermoregulatory changes.

Table III shows the comparison between Gompertz and Logistic curves and it is evident from this Table that both residual sum of squares and absolute percentage variation are less in all environmental conditions for Gompertz curve. The coefficient of determination (R^2%) representing the percentage of variation of the total sum of squares absorbed by the fit of the curves depicted in Table III also indicates that it is higher in

TABLE III : Comparison between Gompertz and logistic curves to estimate plam skin temperature at different environmental conditions.

Environmental conditions	*Fitted curve*	*Coefficient of determination (R^2%)*	*Residual sum of squares*	*Average absolute percentage variation±SEM*
Sea level residents at				
28°C	A	99.47	0.4206	0.3490±0.0704
	B	97.99	1.5860	0.5714±0.1451
21°C	A	99.80	0.1871	0.2062±0.0628
	B	99.37	0.5863	0.3795±0.0955
Stay at high altitude				
Ist week	A	98.91	0.8810	0.5533±0.1198
	B	97.79	1.7831	0.6986±0.1744
IInd week	A	99.19	0.5616	0.4819±0.0871
	B	98.20	0.1262	0.6043±0.1582
IIIrd week	A	99.03	1.1584	0.6890±0.1037
	B	97.88	2.5299	0.7281±0.2362
Return to sea level	A	99.88	0.1439	0.1962±0.0406
	B	99.27	0.8897	0.3852±0.1299
High altitude natives	A	99.85	0.1584	0.2152±0.0530
	B	99.59	0.4238	0.3029±0.0982

A — Gompertz curve
B — Logistic curve

all environmental conditions for Gompertz curve. Thus Gompertz curve describes the above mentioned phenomenon of thermoregulation more efficiently than logistic curve.

DISCUSSION

Attempts have been made on the use of rectangular hyperbola to establish the relationship between pulmonary ventilation and alveolar gas pressure in human subjects (8). Asymptotic curves other than the hyperbola are not frequently used in physiology. These curves were used by several workers in growth studies and sometimes, they were called as growth curves. Katch *et al.* (9) used the linear combination of exponential curves to explain the exercise and recovery pattern of oxygen uptake, ventilation and VE cost by suggesting equation of the form :

$$Yt = C - (a_1 e^{-k} 1^{t} + a_2 e^{-k_2 t})$$

where Yt = the rate of recovery at time 't'

C = resting oxygen consumption,

t = the time after start of exercise.

Studies have been conducted by some workers to see the effect of cold stress imposed on human beings, but the recovery pattern of palm skin temperature after cessation of local cold stress has not been studied statistically. In the present paper Gompertz curve has been used to explain the recovery pattern of palm skin temperature during one such situation more efficiently than Logistic curve. The comparison of these curves (Table III) shows that residual sum of squares and average absolute percentage variation in each environmental condition for Gompertz curve are smaller than that of Logistic curve. It is also evident from Table III that coefficient of determination (R^2%) for Gompertz curve is higher than that of logistic curve in all environmental conditions. The present paper also confirms statistically that thermoregulation efficiency of human subjects during stay at sea level is better than their efficiency at high altitude.

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